

45.

$$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 0, & x = 1 \end{cases}$$

$$f'(x) = \begin{cases} 1, & 0 < x < 1 \\ \times, & x = 1 \end{cases}$$

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4.3 Connecting f' and f'' with the graph of f

first derivative test for local extrema of continuous functions

what does f' tell us about
the graph of f ?

$$\begin{aligned} f' &> 0, & f &\text{inc} \\ f' &< 0, & f &\text{dec} \end{aligned}$$

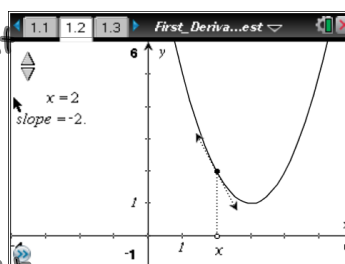
$$\begin{array}{c} f' \quad + \quad 0 \quad - \\ \quad \quad + \quad \times \quad - \end{array} \quad f \text{ has a max}$$

$$\begin{array}{c} f' \quad - \quad 0 \quad + \\ \quad \quad - \quad \times \quad + \end{array} \quad f \text{ has a min}$$

{ for all tests - f must be continuous }

$$\begin{array}{c} f' \quad + \quad 0 \quad + \\ \quad \quad + \quad \times \quad + \end{array}$$

$$\begin{array}{c} f' \quad - \quad 0 \quad - \\ \quad \quad - \quad \times \quad - \end{array}$$



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find the local extrema:

$$y = (x^2 - 3)e^x$$

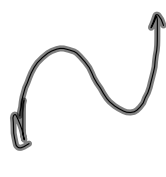
$$y' = (x^2 - 3)e^x + 2xe^x = 0$$

$$y' = e^x(x^2 - 3 + 2x) = e^x(x^2 + 2x - 3) = 0$$

$$y' = e^x(x-1)(x+3) = 0 \quad x = 1, -3$$

$$y' \quad + \quad 0 \quad - \quad 0 \quad +$$

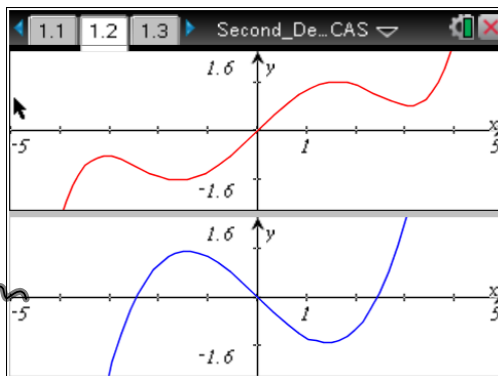
$$\quad \quad \quad -3 \quad \quad \quad 1$$

max at $x = -3$ max is $y = 6e^{-3}$ min at $x = 1$ min is $y = -2e$ 

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concavity test

 $y'' > 0$ y concave up

 $y'' < 0$ y concave down

 inflection pts where y'' changes sign

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Find all points of inflection for the graph of $y = e^{-x^2}$

$$y' = -2xe^{-x^2}$$

$$y'' = -2x(-2xe^{-x^2}) + e^{-x^2} \cdot (-2) = 0$$

$$e^{-x^2} (4x^2 - 2) = 0 \quad x = \pm \sqrt{\frac{1}{2}}$$

$$y'' \begin{matrix} + & 0 & - & 0 & + \\ \hline -\sqrt{\frac{1}{2}} & & & & \sqrt{\frac{1}{2}} \end{matrix}$$

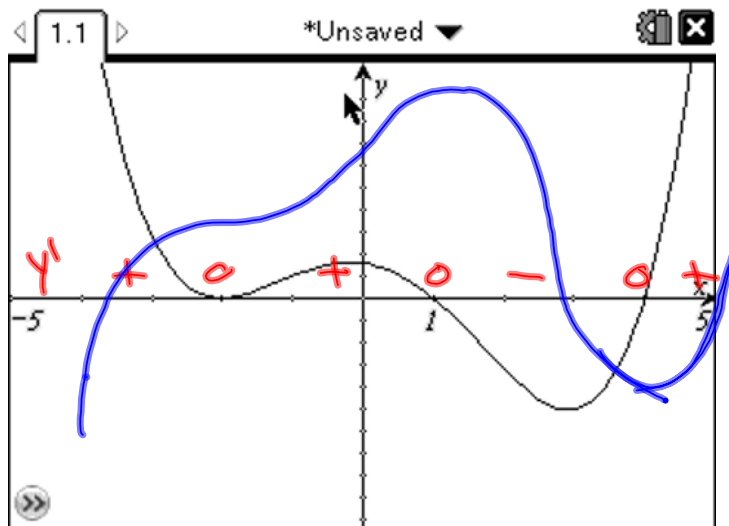
$$4x^2 = 2$$

$$x^2 = \frac{1}{2}$$

inflection points at $x = \pm \sqrt{\frac{1}{2}}$ because y'' changes sign

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This is the graph of f' . Sketch a possible graph of f



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second derivative test for local extrema

if $f' = 0$ & $f'' > 0$ (happy face) then f has a min

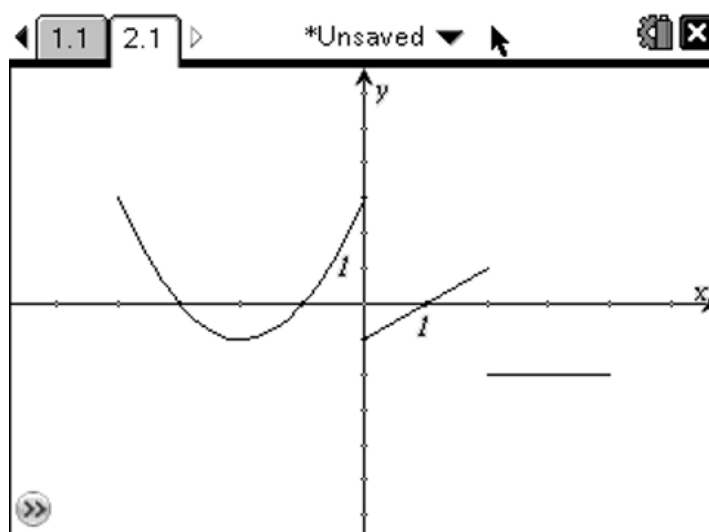
if $f' = 0$ & $f'' < 0$ (sad face) then f has a max

Find the local extrema using the second derivative test

$$y = x^3 - 12x - 5$$

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Given the graph of f' sketch a possible graph of f



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